SAMPLE PAPER

issued by CBSE for Board Exams (2023-24)

Mathematics (041) - Class 12

Time Allowed : 180 Minutes | **Max. Marks : 80**

General Instructions :

1. This Question paper contains **five sections - A, B, C, D and E**. Each section is compulsory. However, there are **internal choices** in some questions.
2. Section A has **18 MCQs** and **02 Assertion-Reason (A-R)** based questions of **1 mark** each.
   Section B has **05 questions** of **2 marks** each.
   Section C has **06 questions** of **3 marks** each.
   Section D has **04 questions** of **5 marks** each.
   Section E has **03 Case-study / Source-based / Passage-based** questions with **sub-parts** (4 **marks** each).
3. There is no overall choice. However, **internal choice** has been provided in
   - **02 Questions of Section B**
   - **03 Questions of Section C**
   - **02 Questions of Section D**
   - **02 Questions of Section E**

   You have to attempt only one of the alternatives in all such questions.

SECTION A

*(Question numbers 01 to 20 carry **1 mark** each.)*

*Followings are **multiple choice questions**. Select the correct option in each one of them.*

01. If $A=[a_{ij}]$ is a square matrix of order 2 such that $a_{ij}=\begin{cases}1, \text{ when } & i\neq j\\ 0, \text{ when } & i=j\end{cases}$, then A^2 is

(a) $\begin{bmatrix}1 & 0\\ 1 & 0\end{bmatrix}$ (b) $\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}$ (c) $\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$ (d) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

02. If A and B are invertible square matrices of the same order, then which of the following is **not** correct?

(a) $\text{adj.}A=|A|A^{-1}$ (b) $\det.(A)^{-1}=[\det.(A)]^{-1}$

(c) $(AB)^{-1}=B^{-1}A^{-1}$ (d) $(A+B)^{-1}=B^{-1}+A^{-1}$

03. If the area of the triangle with vertices (–3, 0), (3, 0) and (0, k) is 9 Sq. units, then the value/s of k will be

(a) 9 (b) ±3 (c) –9 (d) 6

04. If $f(x)=\begin{cases}\dfrac{kx}{|x|}, & \text{if } x<0\\ 3, & \text{if } x\geq 0\end{cases}$ is continuous at $x=0$, then the value of k is

(a) –3 (b) 0 (c) 3 (d) any real number

05. The lines represented by $\vec{r}=\hat{i}+\hat{j}-\hat{k}+\lambda(2\hat{i}+3\hat{j}-6\hat{k})$ and $\vec{r}=2\hat{i}-\hat{j}-\hat{k}+\mu(6\hat{i}+9\hat{j}-18\hat{k})$; (where λ and μ are scalars) are

(a) coincident (b) skew (c) intersecting (d) parallel

06. The degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3=\left(\frac{d^2y}{dx^2}\right)^2$ is

(a) 4 (b) $\frac{3}{2}$ (c) 2 (d) not defined

07. The corner points of the bounded feasible region determined by a system of linear constraints are $(0, 3)$, $(1, 1)$ and $(3, 0)$. Let $Z=px+qy$, where $p, q>0$. The condition on p and q so that the minimum of Z occurs at $(3, 0)$ and $(1, 1)$ is

(a) $p=2q$ (b) $p=\frac{q}{2}$ (c) $p=3q$ (d) $p=q$

08. ABCD is a rhombus whose diagonals intersect at E. Then $\overrightarrow{EA}+\overrightarrow{EB}+\overrightarrow{EC}+\overrightarrow{ED}=$

(a) $\vec{0}$ (b) $\overrightarrow{AD}$ (c) $2\overrightarrow{BD}$ (d) $2\overrightarrow{AD}$

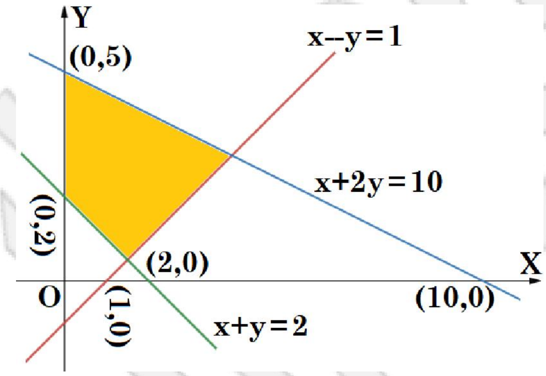
09. For any integer n, the value of $\int_0^{\pi} e^{\sin^2 x}\cos^3(2n+1)x\,dx$ is

(a) -1 (b) 0 (c) 1 (d) 2

10. The value of $|A|$, if $A=\begin{bmatrix}0 & 2x-1 & \sqrt{x}\\ 1-2x & 0 & 2\sqrt{x}\\ -\sqrt{x} & -2\sqrt{x} & 0\end{bmatrix}$, where $x\in R^+$, is

(a) $(2x+1)^2$ (b) 0 (c) $(2x+1)^3$ (d) None of these

11. The feasible region corresponding to the linear constraints of a Linear Programming Problem is given below.



Which of the following is **not** a constraint to the given Linear Programming Problem?

(a) $x+y\ge 2$
(b) $x+2y\le 10$
(c) $x-y\ge 1$
(d) $x-y\le 1$

12. If $\vec{a}=4\hat{i}+6\hat{j}$ and $\vec{b}=3\hat{j}+4\hat{k}$, then the vector form of the component of $\vec{a}$ along $\vec{b}$ is

(a) $\frac{18}{5}(3\hat{i}+4\hat{k})$ (b) $\frac{18}{25}(3\hat{j}+4\hat{k})$ (c) $\frac{18}{5}(3\hat{i}+4\hat{k})$ (d) $\frac{18}{25}(4\hat{i}+6\hat{j})$

13. Given that A is a square matrix of order 3 and $|A|=-2$, then $|adj.(2A)|$ is equal to

(a) -2^6 (b) 4 (c) -2^8 (d) 2^8

14. A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{2},\frac{1}{3},\frac{1}{4}$ respectively. If the events of their solving the problem are independent then the probability that the problem will be solved, is

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

15. The general solution of the differential equation $ydx-xdy=0$; (given $x, y>0$), is of the form

(a) $xy=c$ (b) $x=cy^2$ (c) $y=cx$ (d) $y=cx^2$

16. The value of λ, for which two vectors $2\hat{i}-\hat{j}+2\hat{k}$ and $3\hat{i}+\lambda\hat{j}+\hat{k}$ are perpendicular is

(a) 2 (b) 4 (c) 6 (d) 8

17. The set of all points where the function $f(x)=x+|x|$ is differentiable, is

(a) $(0,\infty)$ (b) $(-\infty,0)$ (c) $(-\infty,0)\cup(0,\infty)$ (d) $(-\infty,\infty)$

18. If the direction cosines of a line are $<\frac{1}{c},\frac{1}{c},\frac{1}{c}>$ then

(a) $0<c<1$ (b) $c>2$ (c) $c=\pm\sqrt{2}$ (d) $c=\pm\sqrt{3}$

Followings are ***Assertion-Reason based questions***.

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).
Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. Let $f(x)$ be a polynomial function of degree 6 such that $\frac{d}{dx}(f(x))=(x-1)^3(x-3)^2$.

Assertion (A) : $f(x)$ has a minimum at $x=1$.

Reason (R) : When $\frac{d}{dx}(f(x))<0,\ \forall x\in(a-h,\ a)$ and $\frac{d}{dx}(f(x))>0,\ \forall x\in(a,\ a+h)$; where 'h' is an infinitesimally small positive quantity, then $f(x)$ has a minimum at $x=a$, provided $f(x)$ is continuous at $x=a$.

20. **Assertion (A) :** The relation $f:\{1,2,3,4\}\to\{x,y,z,p\}$ defined by $f=\{(1,x),(2,y),(3,z)\}$ is a bijective function.

Reason (R) : The function $f:\{1,2,3\}\to\{x,y,z,p\}$ such that $f=\{(1,x),(2,y),(3,z)\}$ is a one-one function.

SECTION B

(Question numbers 21 to 25 carry ***2 marks*** *each.)*

21. Find the value of $\sin^{-1}\left(\cos\left(\frac{33\pi}{5}\right)\right)$.

OR

Find the domain of $\sin^{-1}(x^2-4)$.

22. Find the interval/s in which the function $f:R\to R$ defined by $f(x)=xe^x$, is increasing.

23. If $f(x)=\frac{1}{4x^2+2x+1}$; $x\in R$, then find the maximum value of $f(x)$.

OR

Find the maximum profit that a company can make, if the profit function is given by $P(x)=72+42x-x^2$, where x is the number of units and P is the profit in rupees.

24. Evaluate : $\int_{-1}^{1}\log_e\left(\frac{2-x}{2+x}\right)dx$.

25. Check whether the function $f:R\to R$ defined by $f(x)=x^3+x$, has any critical point/s or not? If yes, then find the point/s.

SECTION C

(Question numbers 26 to 31 carry ***3 marks*** *each.)*

26. Evaluate : $\int \frac{2x^2+3}{x^2(x^2+9)}dx;\ x \neq 0$.

27. The random variable X has a probability distribution P(X) of the following form, where 'k' is some real number:

$$P(X)=\begin{cases} k, \text{ if } x=0 \\ 2k, \text{ if } x=1 \\ 3k, \text{ if } x=2 \\ 0, \text{ otherwise} \end{cases}.$$

(i) Determine the value of k.

(ii) Find $P(X<2)$.

(iii) Find $P(X>2)$.

28. Evaluate : $\int \sqrt{\frac{x}{1-x^3}}\,dx;\ x \in (0, 1)$.

OR

Evaluate : $\int_0^{\frac{\pi}{4}} \log_e(1+\tan x)\,dx$.

29. Solve the differential equation : $ye^{\frac{x}{y}}dx = \left(xe^{\frac{x}{y}} + y^2\right)dy,\ (y \neq 0)$.

OR

Solve the differential equation : $(\cos^2 x)\frac{dy}{dx} + y = \tan x;\ \left(0 \le x \le \frac{\pi}{2}\right)$.

30. Solve the following Linear Programming graphically.

Minimize $z = x + 2y$.

Subject to the constraints $x+2y \ge 100,\ 2x-y \le 0,\ 2x+y \le 200,\ x, y \ge 0$.

OR

Solve the following Linear Programming graphically.

Maximize $z = -x + 2y$.

Subject to the constraints $x \ge 3,\ x+y \ge 5,\ x+2y \ge 6,\ y \ge 0$.

31. If $(a+bx)e^{\frac{y}{x}} = x$, then prove that $x\frac{d^2y}{dx^2} = \left(\frac{a}{a+bx}\right)^2$.

SECTION D

(Question numbers 32 to 35 carry ***5 marks*** *each.)*

32. Make a rough sketch of the region $\{(x, y): 0 \le y \le x^2+1,\ 0 \le y \le x+1,\ 0 \le x \le 2\}$ and find the area of the region, using the method of integration.

33. Let N be the set of all natural numbers and R be a relation on $N \times N$ defined by $(a, b)R(c, d) \Leftrightarrow ad = bc$ for all $(a, b), (c, d) \in N \times N$.

Show that R is an equivalence relation on $N \times N$.

Also, find the equivalence class of (2, 6), i.e., [(2, 6)].

OR

Show that the function $f : R \to \{x \in R : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in R$ is one-one and onto function.

34. Using the matrix method, solve the following system of linear equations :

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4,\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1,\ \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2.$$

35. Find the coordinates of the image of the point $(1, 6, 3)$ with respect to the line $\vec{r}=(\hat{j}+2\hat{k})+\lambda(\hat{i}+2\hat{j}+3\hat{k})$; where '$\lambda$' is a scalar.
Also, find the distance of the image from the y-axis.

OR

An aeroplane is flying along the line $\vec{r}=\lambda(\hat{i}-\hat{j}+\hat{k})$; where '$\lambda$' is a scalar and another aeroplane is flying along the line $\vec{r}=\hat{i}-\hat{j}+\mu(-2\hat{j}+\hat{k})$; where '$\mu$' is a scalar. At what points on the lines should they reach, so that the distance between them is the shortest? Find the shortest possible distance between them.

SECTION E

*(Question numbers 36 to 38 carry **4 marks** each.)*
*This section contains **three Case-study / Passage based questions**.*
*First two questions have **three sub-parts** (i), (ii) and (iii) of **marks 1, 1 and 2** respectively.*
*Third question has **two sub-parts** of **2 marks** each.*

36. **CASE STUDY I :** Read the following passage and then answer the questions given below.
In an office three employees James, Sophia and Oliver process incoming copies of a certain form. James processes 50% of the forms, Sophia processes 20% and Oliver the remaining 30% of the forms. James has an error rate of 0.06, Sophia has an error rate of 0.04 and Oliver has an error rate of 0.03.

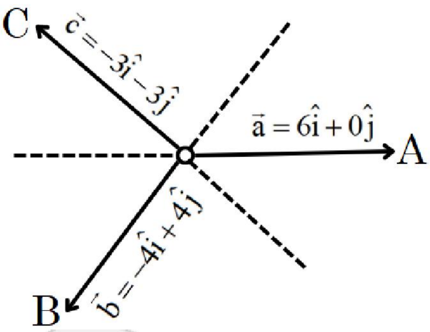
(i) Find the probability that Sophia processed the form and committed an error.
(ii) Find the total probability of committing an error in processing the form.
(iii) The manager of the Company wants to do a quality check. During inspection, he selects a form at random from the days output of processed form. If the form selected at random has an error, find the probability that the form is not processed by James.

OR

(iii) Let E be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that James, Sophia and Oliver processed the form. Find the value of $\sum_{i=1}^{3} P(E_i|E)$.

37. **CASE STUDY II :** Read the following passage and then answer the questions given below.

Teams A, B and C went for playing a tug of war game. Teams A, B and C have attached a rope to a metal ring and are trying to pull the ring into their own area.



Team A pulls with force $F_1 = 6\hat{i} + 0\hat{j}$ kN.

Team B pulls with force $F_2 = -4\hat{i} + 4\hat{j}$ kN.

Team C pulls with force $F_3 = -3\hat{i} - 3\hat{j}$ kN.

(i) What is the magnitude of the force of Team A?

(ii) Which team will win the game?

(iii) Find the magnitude of the resultant force exerted by the teams.

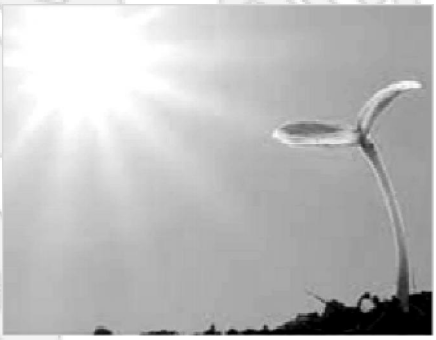
OR

(iii) In what direction is the ring getting pulled?

38. **CASE STUDY III :** Read the following passage and then answer the questions given below.

The relation between the height of the plant ('y' in cm) with respect to its exposure to the sunlight is governed by the following equation

$y = 4x - \frac{1}{2}x^2$, where 'x' is the number of days exposed to the sunlight, for $x \le 3$.



(i) Find the rate of growth of the plant with respect to the number of days exposed to the sunlight.

(ii) Does the rate of growth of the plant increase or decrease in the first three days? What will be the height of the plant after 2 days?

Detailed Solutions for CBSE Sample Paper (2023-24)

SECTION A

01. (d) Note that $A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ $\therefore A^2 = A.A = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$.

02. (d) The statement "$(A+B)^{-1} = B^{-1} + A^{-1}$" is not correct.

03. (b) Area = Magnitude of $\frac{1}{2}\begin{vmatrix}-3 & 0 & 1\\ 3 & 0 & 1\\ 0 & k & 1\end{vmatrix}$

$\Rightarrow \pm 9 = \frac{1}{2}\begin{vmatrix}-3 & 0 & 1\\ 3 & 0 & 1\\ 0 & k & 1\end{vmatrix}$

Expanding along C_2, we get $\pm 18 = -0+0-k(-3-3)$ $\Rightarrow k=\pm 3$.

04. (a) Since f is continuous at $x=0$.

Therefore, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$

$\Rightarrow \lim_{x\to 0^-} \frac{-kx}{x} = \lim_{x\to 0^+} 3 = 3$

$\Rightarrow \lim_{x\to 0^-} (-k) = 3$

$\Rightarrow (-k) = 3$ $\therefore k=-3$.

05. (d) Note that $6\hat{i}+9\hat{j}-18\hat{k} = 3(2\hat{i}+3\hat{j}-6\hat{k})$.

That means, $2\hat{i}+3\hat{j}-6\hat{k}$ and $6\hat{i}+9\hat{j}-18\hat{k}$ are parallel.

Also the fixed point $\hat{i}+\hat{j}-\hat{k}$ on the line $\vec{r} = \hat{i}+\hat{j}-\hat{k}+\lambda(2\hat{i}+3\hat{j}-6\hat{k})$ does not satisfy $\vec{r} = 2\hat{i}-\hat{j}-\hat{k}+\mu(6\hat{i}+9\hat{j}-18\hat{k})$; where λ and μ are scalars.

06. (c) For the D.E. $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$, the higher order derivative is $\frac{d^2y}{dx^2}$.

Clearly the degree is 2.

07. (b) $Z = px+qy$

At $(3, 0)$, $Z = 3p$ ...(i)

At $(1, 1)$, $Z = p+q$ ...(ii)

From (i) and (ii), $3p = p+q$

$\Rightarrow 2p = q$ $\therefore p = \frac{q}{2}$.

08. (a) ABCD is a rhombus whose diagonals bisect each other. Consider the diagram.

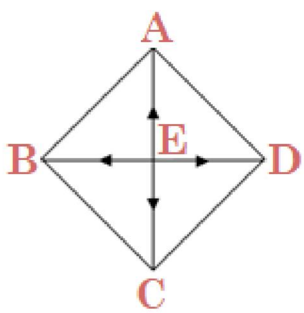
That is, $|\overrightarrow{EA}| = |\overrightarrow{EC}|$ and $|\overrightarrow{EB}| = |\overrightarrow{ED}|$.

But since they are opposite to each other so, they are of opposite signs.

That is, $\overrightarrow{EA} = -\overrightarrow{EC}$ and $\overrightarrow{EB} = -\overrightarrow{ED}$.

$\Rightarrow \overrightarrow{EA}+\overrightarrow{EC} = \vec{0}$ ...(i)

and $\overrightarrow{EB}+\overrightarrow{ED} = \vec{0}$ ...(ii)



Adding (i) and (ii), we get $\overrightarrow{EA}+\overrightarrow{EB}+\overrightarrow{EC}+\overrightarrow{ED} = \vec{0}$.

09. (b) Let $f(x)=e^{\sin^2 x}\cos^3(2n+1)x$

$\because f(\pi-x)=e^{\sin^2(\pi-x)}\cos^3(2n+1)(\pi-x)=-e^{\sin^2 x}\cos^3(2n+1)x=-f(x)$

$\therefore \int_0^{\pi} e^{\sin^2 x}\cos^3(2n+1)x\,dx=0.$

Recall that, if f is integrable in $[0, 2a]$ and $f(2a-x)=-f(x)$, then $\int_0^{2a} f(x)\,dx=0$.

10. (b) Matrix A is a skew symmetric matrix of odd order (order of A is 3) $\therefore |A|=0$.

11. (c) We observe that $(0, 0)$ does not satisfy the inequality $x-y\ge 1$.
So, the half plane represented by $x-y\ge 1$ will not contain origin therefore, it will not contain the shaded feasible region.

12. (b) Vector component of $\vec{a}$ along $\vec{b}=(\vec{a}.\hat{b})\hat{b}=\left(\dfrac{\vec{a}.\vec{b}}{|\vec{b}|^2}\right)\vec{b}=\dfrac{18}{25}(3\hat{j}+4\hat{k})$.

13. (d) $|adj.(2A)|=|(2A)|^2=(2^3|A|)^2=2^6|A|^2=2^6\times(-2)^2=2^8$.

14. (d) Let A, B, C be the respective events of solving the problem by three students.

Then $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{3}$ and $P(C)=\frac{1}{4}$ $\therefore P(\overline{A})=\frac{1}{2}$, $P(\overline{B})=\frac{2}{3}$ and $P(\overline{C})=\frac{3}{4}$.

Here A, B, C are independent events.

$\because$ Problem is solved if at least one of them solves the problem.

$\therefore$ Required probability is $=P(A\cup B\cup C)=1-P(\overline{A})P(\overline{B})P(\overline{C})$

$$=1-\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}=1-\frac{1}{4}=\frac{3}{4}.$$

Alternatively,

The problem will be solved if one or more of them can solve the problem.

Therefore, required probability is

$$P(A\overline{B}\overline{C})+P(\overline{A}B\overline{C})+P(\overline{A}\overline{B}C)+P(AB\overline{C})+P(A\overline{B}C)+P(\overline{A}BC)+P(ABC)$$

$$=\frac{1}{2}.\frac{2}{3}.\frac{3}{4}+\frac{1}{2}.\frac{1}{3}.\frac{3}{4}+\frac{1}{2}.\frac{2}{3}.\frac{1}{4}+\frac{1}{2}.\frac{1}{3}.\frac{3}{4}+\frac{1}{2}.\frac{2}{3}.\frac{1}{4}+\frac{1}{2}.\frac{1}{3}.\frac{1}{4}+\frac{1}{2}.\frac{1}{3}.\frac{1}{4}=\frac{3}{4}.$$

15. (c) $ydx-xdy=0$

$\Rightarrow ydx=xdy$

$\Rightarrow \dfrac{dy}{y}=\dfrac{dx}{x}$

On integrating, we get $\int\dfrac{dy}{y}=\int\dfrac{dx}{x}$

$\Rightarrow \log_e|y|=\log_e|x|+\log_e|c|$

Since $x, y, c>0$, we write $\log_e y=\log_e x+\log_e c$

$\Rightarrow \log_e y=\log_e(cx)$

$\Rightarrow y=cx$.

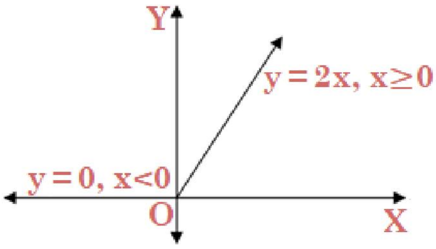
16. (d) Since Dot product of two perpendicular vectors is zero.

$\therefore (2\hat{i}-\hat{j}+2\hat{k}).(3\hat{i}+\lambda\hat{j}+\hat{k})=0$

$\Rightarrow 2\times3+(-1)\lambda+2\times1=0$

$\Rightarrow \lambda=8$.

17. (c) $f(x)=x+|x|=\begin{cases}2x\,,\ x\ge 0\\ 0,\ x<0\end{cases}$



Let $y=f(x)$

Consider the diagram (graph) shown.

There is a sharp corner at $x=0$, so $f(x)$ is not differentiable at $x=0$.

Alternatively,

$f(x)=x+|x|=\begin{cases}2x\,,\ x\ge 0\\ 0,\ x<0\end{cases} \qquad \Rightarrow f'(x)=\begin{cases}2,\ x\ge 0\\ 0,\ x<0\end{cases}$

$\because Lf'(0)=0$ and $Rf'(0)=2$

$\therefore$ Function f is not differentiable at $x=0$.

For $x\ge 0$, $f(x)=2x$ (linear function); when $x<0$, $f(x)=0$ (constant function).

Hence, $f(x)$ is differentiable only when $x\in(-\infty,\,0)\cup(0,\,\infty)$.

18. (d) We know that, $l^2+m^2+n^2=1$

$\Rightarrow \left(\dfrac{1}{c}\right)^2+\left(\dfrac{1}{c}\right)^2+\left(\dfrac{1}{c}\right)^2=1$

$\Rightarrow 3\left(\dfrac{1}{c}\right)^2=1 \quad \Rightarrow c^2=3$

$\Rightarrow c=\pm\sqrt{3}.$

19. (a) Given $\dfrac{d}{dx}\big(f\,(x)\big)=(x-1)^3(x-3)^2$

Note that $\dfrac{d}{dx}\big(f\,(x)\big)<0,\ \forall\, x\in(1-h,\,1)$

and $\dfrac{d}{dx}\big(f\,(x)\big)>0,\ \forall\, x\in(1,\,1+h)$

Clearly, **A** and **R** both are true, also **R** is correct explanation of **A**.

20. (d) **A** is false. Since the element 4 has no image under f. So the relation f is not a function. That means, f can't be a bijective function.

Moreover, **R** is true. The given function f is one-one, because for each element $\in\{1,\,2,\,3\}$, there is a different image in $\{x,\,y,\,z,\,p\}$ under f.

SECTION B

21. $\sin^{-1}\left(\cos\left(\dfrac{33\pi}{5}\right)\right)=\sin^{-1}\cos\left(6\pi+\dfrac{3\pi}{5}\right)=\sin^{-1}\cos\left(\dfrac{3\pi}{5}\right)=\dfrac{\pi}{2}-\cos^{-1}\cos\left(\dfrac{3\pi}{5}\right)$

$=\dfrac{\pi}{2}-\dfrac{3\pi}{5}=-\dfrac{\pi}{10}.$

OR

For $\sin^{-1}(x^2-4)$ to be defined, we must have $-1\le(x^2-4)\le 1$

$\Rightarrow 3\le x^2\le 5$

$\Rightarrow \sqrt{3}\le|x|\le\sqrt{5}$

$\Rightarrow x\in\left[-\sqrt{5},\,-\sqrt{3}\right]\cup\left[\sqrt{3},\,\sqrt{5}\right].$

So, domain is $\left[-\sqrt{5},\,-\sqrt{3}\right]\cup\left[\sqrt{3},\,\sqrt{5}\right].$

22. $f(x)=xe^x$

$\Rightarrow f'(x)=e^x(x+1)$

For $f'(x)=e^x(x+1)=0$, we get $x=-1$

When $x\in[-1,\infty)$, $(x+1)\geq 0$ and $e^x>0$

$\Rightarrow f'(x)\geq 0$

$\therefore f(x)$ increases in $x\in[-1,\infty)$.

23. Given $f(x)=\dfrac{1}{4x^2+2x+1}$

Let $g(x)=\dfrac{1}{f(x)}=4x^2+2x+1$

$\Rightarrow g(x)=4\left(x^2+2x\dfrac{1}{4}+\dfrac{1}{16}\right)+\dfrac{3}{4}$

$\Rightarrow g(x)=4\left(x+\dfrac{1}{4}\right)^2+\dfrac{3}{4}\geq\dfrac{3}{4}$

$\therefore$ Maximum value of $f(x)=\dfrac{4}{3}$.

Alternatively,

Given $f(x)=\dfrac{1}{4x^2+2x+1}$

Let $g(x)=\dfrac{1}{f(x)}=4x^2+2x+1$

$\Rightarrow g'(x)=8x+2$ and $g''(x)=8$

For $g'(x)=0,\ 8x+2=0 \quad \Rightarrow x=-\dfrac{1}{4}$

$\because g''\left(x=-\dfrac{1}{4}\right)=8>0$

$\therefore g(x)$ is minimum when $x=-\dfrac{1}{4}$.

So, $f(x)$ is maximum at $x=-\dfrac{1}{4}$.

$\therefore$ Maximum value of $f(x)=f\left(-\dfrac{1}{4}\right)=\dfrac{1}{4\left(-\dfrac{1}{4}\right)^2+2\left(-\dfrac{1}{4}\right)+1}=\dfrac{4}{3}$.

Note that, if you do not take $g(x)=\dfrac{1}{f(x)}$ and directly proceed with differentiation of $f(x)$ then too this problem can be solved.

OR

Given $P(x)=72+42x-x^2$, where profit P is in the rupees (₹).

$\therefore P'(x)=42-2x$ and $P''(x)=-2$

For maxima and minima, $P'(x)=0$, $42-2x=0 \qquad \Rightarrow x=21$

$\because P''(21)=-2<0$

So, $P(x)$ is maximum at $x=21$.

The maximum value of $P(x) = P(21) = 72 + (42 \times 21) - (21)^2 = 513$.
Therefore, the maximum profit is ₹513.

24. Let $f(x) = \log_e\left(\frac{2-x}{2+x}\right)$

Note that $f(-x) = \log_e\left(\frac{2+x}{2-x}\right) = -\log_e\left(\frac{2-x}{2+x}\right) = -f(x)$

That means, $f(x)$ is an odd function.

$\therefore \int_{-1}^{1} \log_e\left(\frac{2-x}{2+x}\right)dx = 0$.

Recall that, if f is integrable in [–a, a] and $f(-x) = -f(x)$, then $\int_{-a}^{a} f(x)\,dx = 0$.

25. $f(x) = x^3 + x$, for all $x \in R$.

$\therefore f'(x) = 3x^2 + 1$

Since for all $x \in R$, $x^2 \ge 0$

$\therefore f'(x) > 0$

Hence, no critical point exists for $f(x)$.

SECTION C

26. Take $x^2 = t$.

Then $\frac{2x^2+3}{x^2(x^2+9)} = \frac{2t+3}{t(t+9)} = \frac{A}{t} + \frac{B}{t+9}$

$\Rightarrow 2t + 3 = A(t+9) + Bt$

On comparing both sides, we get $9A = 3$, $A + B = 2$

On solving, we get $A = \frac{1}{3}$ and $B = \frac{5}{3}$.

Now $\int \frac{2x^2+3}{x^2(x^2+9)}dx = \frac{1}{3}\int \frac{dx}{x^2} + \frac{5}{3}\int \frac{dx}{x^2+9} = -\frac{1}{3x} + \frac{5}{9}\tan^{-1}\left(\frac{x}{3}\right) + c$.

27. (i) Recall that $\sum P(X = r) = 1$

$\Rightarrow P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + ... = 1$

$\Rightarrow k + 2k + 3k + 0 + 0 + ... = 1$

$\Rightarrow k = \frac{1}{6}$.

(ii) $P(X<2) = P(X=0) + P(X=1) = k + 2k = 3k = 3 \times \frac{1}{6} = \frac{1}{2}$.

(iii) $P(X>2) = P(X=3) + P(X=4) + ...$

$\therefore P(X>2) = 0$.

28. Let $x^{\frac{3}{2}} = t \Rightarrow \sqrt{x}\,dx = \frac{2}{3}dt$.

Now $\int \sqrt{\frac{x}{1-x^3}}dx = \int \frac{\sqrt{x}}{\sqrt{1-(x^{3/2})^2}}dx = \frac{2}{3}\int \frac{dt}{\sqrt{1-t^2}}$

$= \frac{2}{3}\sin^{-1}(t) + c = \frac{2}{3}\sin^{-1}\left(x^{\frac{3}{2}}\right) + c$.

OR

Let $I=\int_0^{\frac{\pi}{4}}\log_e(1+\tan x)\,dx$ …(i)

$\Rightarrow I=\int_0^{\frac{\pi}{4}}\log_e\left(1+\tan\left(\frac{\pi}{4}-x\right)\right)dx$ (Using $\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$

$\Rightarrow I=\int_0^{\frac{\pi}{4}}\log_e\left(1+\frac{1-\tan x}{1+\tan x}\right)dx=\int_0^{\frac{\pi}{4}}\log_e\left(\frac{2}{1+\tan x}\right)dx$

$\Rightarrow I=\int_0^{\frac{\pi}{4}}\log_e 2dx-\int_0^{\frac{\pi}{4}}\log_e(1+\tan x)dx$

$\Rightarrow I=\int_0^{\frac{\pi}{4}}\log_e 2dx-I$ (Using (i)

$\Rightarrow 2I=\log_e 2[x]_0^{\pi/4}$

$\Rightarrow 2I=\log_e 2\left[\frac{\pi}{4}-0\right]$

$\Rightarrow I=\frac{\pi}{8}\log_e 2.$

29. $ye^{\frac{x}{y}}dx=\left(xe^{\frac{x}{y}}+y^2\right)dy$

$\Rightarrow \frac{dx}{dy}=\frac{xe^{\frac{x}{y}}+y^2}{ye^{\frac{x}{y}}}$

$\Rightarrow \frac{dx}{dy}=\frac{x}{y}+\frac{y}{e^{\frac{x}{y}}}$ …(i)

Put $x=vy\Rightarrow\frac{dx}{dy}=v+y\frac{dv}{dy}$.

So equation (i) becomes $v+y\frac{dv}{dy}=v+\frac{y}{e^v}$

$\Rightarrow y\frac{dv}{dy}=\frac{y}{e^v}$ $\Rightarrow e^v dv=dy$

On integrating, we get $\int e^v dv=\int dy$

$\Rightarrow e^v=y+c$

$\Rightarrow e^{x/y}=y+c$.

OR

$(\cos^2 x)\frac{dy}{dx}+y=\tan x$

Dividing both the sides by $\cos^2 x$, we get $\frac{dy}{dx}+\frac{y}{\cos^2 x}=\frac{\tan x}{\cos^2 x}$

$\frac{dy}{dx} + y(\sec^2 x) = \tan x \sec^2 x \quad \ldots(i)$

Comparing with $\frac{dy}{dx} + P(x)y = Q(x)$, we get $P(x) = \sec^2 x,\ Q(x) = \tan x \sec^2 x$

The integrating factor will be, $I.F. = e^{\int \sec^2 x\, dx} = e^{\tan x}$

Required solution is given as $y(e^{\tan x}) = \int (e^{\tan x}) \tan x \sec^2 x dx + c$

Put $\tan x = t \Rightarrow \sec^2 x dx = dt$ in the integral in RHS.

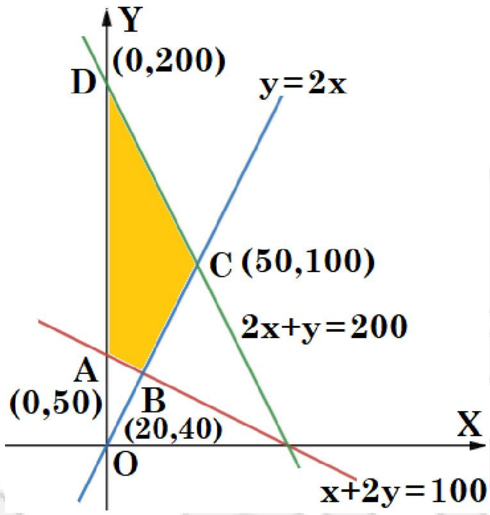
$\therefore y e^{\tan x} = \int t e^t dt + c$

$\Rightarrow y e^{\tan x} = t e^t - e^t + c$

$\Rightarrow y e^{\tan x} = (\tan x - 1) e^{\tan x} + c$

$\therefore y = (\tan x - 1) + c e^{-\tan x}$.

30. Graph with the feasible region for the given constraints is given below.

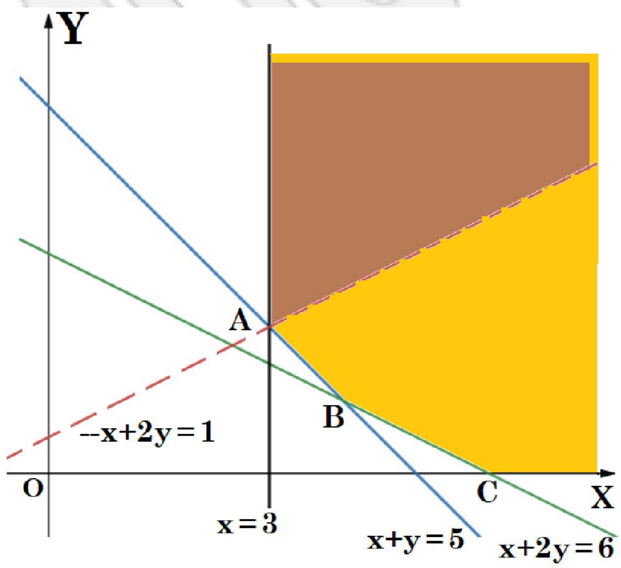


Corner point	Value of Z	
A(0, 50)	100	Minimum
B(20, 40)	100	Minimum
C(50, 100)	250	
D(0, 200)	400	

The minimum value of Z is 100, at all the points on the line segment joining the points (0, 50) and (20, 40).

OR

Consider the graph shown with feasible region for the given constraints.
Note that the corner points are $A(3, 2)$, $B(4, 1)$ and $C(6, 0)$.



Corner point	Value of Z	
A(3, 2)	1	Maximum
B(4, 1)	–2	
C(6, 0)	–6	

Observe that the feasible region obtained is unbounded.
That means, $Z = 1$ may or may not be the maximum value.
To check, let $-x + 2y > 1$.
It is clearly evident that the resulting open half-plane $-x + 2y > 1$ has points in common with the feasible region.
Hence, $Z = 1$ is **not** the maximum value.
We conclude, Z has **no** maximum value.

31. $(a+bx)e^{y/x}=x \quad \Rightarrow e^{y/x}=\dfrac{x}{a+bx}$

On taking logarithm both the sides, we get $\dfrac{y}{x}=\log_e\left(\dfrac{x}{a+bx}\right)=\log_e x-\log_e(a+bx)$

On differentiating with respect to x, $\dfrac{x\dfrac{dy}{dx}-y}{x^2}=\dfrac{1}{x}-\dfrac{1}{a+bx}\times\dfrac{d}{dx}(a+bx)$

$\Rightarrow \dfrac{x\dfrac{dy}{dx}-y}{x^2}=\dfrac{1}{x}-\dfrac{b}{a+bx}$

$\Rightarrow x\dfrac{dy}{dx}-y=x^2\left(\dfrac{1}{x}-\dfrac{b}{a+bx}\right)=\dfrac{ax}{a+bx}$

On differentiating again with respect to x, $x\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}-\dfrac{dy}{dx}=\dfrac{(a+bx)a-ax(b)}{(a+bx)^2}$

$\Rightarrow x\dfrac{d^2y}{dx^2}=\left(\dfrac{a}{a+bx}\right)^2$.

SECTION D

32. Consider $y=x^2+1$, $y=x+1$.

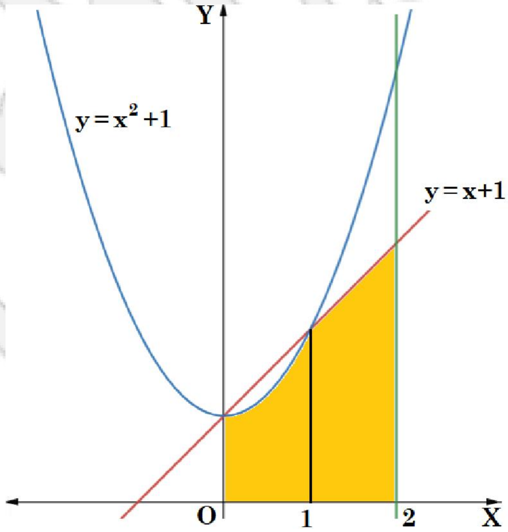
We need to find the point of intersection of the curve $y=x^2+1$ and the line $y=x+1$.

We write $x^2+1=x+1$

$\Rightarrow x(x-1)=0$

$\Rightarrow x=0,1$.

So, the point of intersections are (0, 1) and (1, 2).



Required area $=\int_0^1 (x^2+1)\,dx+\int_1^2 (x+1)\,dx$

$=\left[\dfrac{x^3}{3}+x\right]_0^1+\left[\dfrac{x^2}{2}+x\right]_1^2$

$=\left[\left(\dfrac{1}{3}+1\right)-0\right]+\left[(2+2)-\left(\dfrac{1}{2}+1\right)\right]$

$=\dfrac{23}{6}$ Sq. units.

33. Let (a, b) be an arbitrary element of $N\times N$. Then, $(a, b)\in N\times N$ and $a, b\in N$.

We have, $ab=ba$.

$\because a, b\in N$ and the multiplication is commutative on N.

$\Rightarrow (a, b)\,R\,(a, b)$, according to the definition of the relation R on $N\times N$.

Thus $(a, b)\,R\,(a, b)\ \forall (a, b)\in N\times N$.

So, R is reflexive relation on $N\times N$.

Let $(a, b), (c, d)$ be arbitrary elements of $N\times N$ such that $(a, b)\,R\,(c, d)$.

Then, $(a, b)\,R\,(c, d) \quad \Rightarrow ad=bc$

$\Rightarrow bc=ad$

$\Rightarrow cb=da$

$\Rightarrow (c, d)\,R\,(a, b)$

Thus, $(a, b)R(c, d) \Rightarrow (c, d)R(a, b)$

So, R is symmetric relation on $N \times N$.

Let $(a, b), (c, d), (e, f)$ be arbitrary elements of $N \times N$ such that $(a, b)R(c, d)$ and $(c, d)R(e, f)$.

Thus, $(a, b)R(c, d) \Rightarrow ad = bc$ and $(c, d)R(e, f) \Rightarrow cf = de$

Consider $(ad)(cf) = (bc)(de)$

$\Rightarrow af = be$

$\Rightarrow (a, b)R(e, f)$

Thus $(a, b)R(c, d)$ and $(c, d)R(e, f) \Rightarrow (a, b)R(e, f)$

So, R is transitive relation on $N \times N$.

As the relation R is reflexive, symmetric and transitive so, it is equivalence relation on $N \times N$.

$[(2, 6)] = \{(x, y) \in N \times N : (x, y)R(2, 6)\}$

$= \{(x, y) \in N \times N : 6x = 2y\}$

$= \{(x, y) \in N \times N : 3x = y\}$

$= \{(x, 3x) : x \in N\}$

$= \{(1, 3), (2, 6), (3, 9), ...\}$.

OR

Here $f : R \to A$, where $A = \{x \in R : -1 < x < 1\}$, is defined by $f(x) = \dfrac{x}{1+|x|}$, $x \in R$.

One-one : Let $x_1, x_2 \in R$.

Also let $f(x_1) = f(x_2)$.

That is, $\dfrac{x_1}{1+|x_1|} = \dfrac{x_2}{1+|x_2|}$

Case I : If $x_1, x_2 > 0$ then, $\dfrac{x_1}{1+x_1} = \dfrac{x_2}{1+x_2}$

$\Rightarrow x_1 + x_1x_2 = x_2 + x_1x_2$

$\Rightarrow x_1 = x_2 \ldots$(i)

Case II : If $x_1, x_2 < 0$ then, $\dfrac{x_1}{1-x_1} = \dfrac{x_2}{1-x_2}$

$\Rightarrow x_1 - x_1x_2 = x_2 - x_1x_2$

$\Rightarrow x_1 = x_2 \ldots$(ii)

Case III : If $x_1 > 0, x_2 < 0$ then, clearly $x_1 \neq x_2$. Therefore, $\dfrac{x_1}{1+x_1} \neq \dfrac{x_2}{1-x_2}$

$\Rightarrow f(x_1) \neq f(x_2) \ldots$(iii)

Case IV : If $x_1 < 0, x_2 > 0$ then, clearly $x_1 \neq x_2$. Therefore, $\dfrac{x_1}{1-x_1} \neq \dfrac{x_2}{1+x_2}$

$\Rightarrow f(x_1) \neq f(x_2) \ldots$(iv)

By (i), (ii), (iii) and (iv), it is evident that the function f is one-one.

Onto : Let $y \in A$ $\therefore -1 < y < 1$ so that $y = f(x)$.

Recall that, $A = \{x \in R : -1 < x < 1\}$.

Now $y = \dfrac{x}{1+|x|}$ $\qquad \Rightarrow y = \dfrac{x}{1 \pm x}$

$\Rightarrow y \pm xy = x$
$\Rightarrow y = x \mp xy$
$\Rightarrow y = x(1 \mp y)$

$\Rightarrow x = \dfrac{y}{1 \mp y} \in R$ for all $-1 < y < 1$.

That is, for all f-image in the Codomain A, we've a pre-image in the Domain R of the function f.
So, f is onto function.

34. The given system of equations can be written in the form $AX = B$, where

$$A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}.$$

Now $|A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix} = 1200 \neq 0 \quad \therefore A^{-1}$ exists.

$$\therefore \text{adj.} A = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

Hence, $A^{-1} = \dfrac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$

Since $AX = B$

$\Rightarrow A^{-1}AX = A^{-1}B$

$\Rightarrow IX = A^{-1}B$

$\Rightarrow X = A^{-1}B$

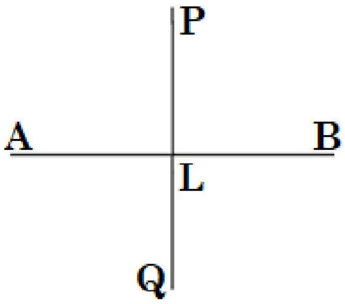
$$\Rightarrow X = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \frac{1}{1200} \begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

Thus, $\dfrac{1}{x} = \dfrac{1}{2}, \dfrac{1}{y} = \dfrac{1}{3}, \dfrac{1}{z} = \dfrac{1}{5}$

Hence, $x = 2,\ y = 3,\ z = 5$.

35. Let $P(1, 6, 3)$ be the given point, and let L be the foot of perpendicular from P to the given line AB (shown in the figure).



Let $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}=\lambda$.

The coordinates of a general point on the given line AB are given by $x=\lambda$, $y=2\lambda+1$ and $z=3\lambda+2$.

Let point L be given by $(\lambda,\ 2\lambda+1,\ 3\lambda+2)$.

So, direction ratios of PL are $\lambda-1,\ \ 2\lambda+1-6$ and $3\lambda+2-3$

That is, $\lambda-1,\ 2\lambda-5$ and $3\lambda-1$.

$\because$ Direction ratios of the given line are 1, 2, and 3; also line AB is perpendicular to PL.

$\therefore\ (\lambda-1)(1)+(2\lambda-5)(2)+(3\lambda-1)(3)=0$ (Using $a_1a_2+b_1b_2+c_1c_2=0$

$\Rightarrow \lambda=1$

So, coordinates of L are (1, 3, 5).

Let $Q(x_1,\ y_1,\ z_1)$ be the image of $P(1,\ 6,\ 3)$ in the given line.

Then, L is the mid-point of PQ.

So, $\frac{x_1+1}{2}=1,\ \frac{y_1+6}{2}=3$ and $\frac{z_1+3}{2}=5$

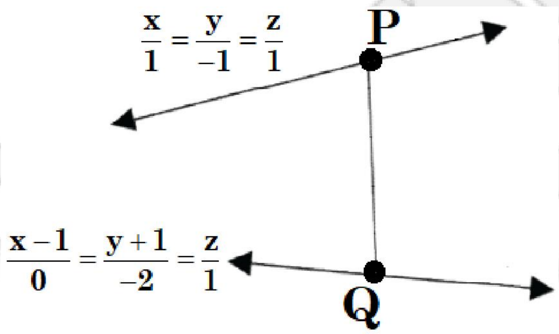
$\Rightarrow x_1=1,\ y_1=0$ and $z_1=7$

Hence, the image of $P(1,\ 6,\ 3)$ in the given line AB is $Q(1,\ 0,\ 7)$.

Now, the distance of the point $Q(1,\ 0,\ 7)$ from the y-axis is $\sqrt{1^2+7^2}=\sqrt{50}$ units.

OR

Consider the following diagram.



The equation of two given straight lines in the Cartesian form are $\frac{x}{1}=\frac{y}{-1}=\frac{z}{1}\ \ldots$(i) and

$\frac{x-1}{0}=\frac{y+1}{-2}=\frac{z}{1}\ \ldots$(ii)

Note that the lines are not parallel as their direction ratios are not proportional.

Let P be a point on the line (i) and Q be a point on the line (ii) such that line PQ is perpendicular to both of the lines.

Let $P(\lambda,-\lambda,\ \lambda)$ be any random point on the line (i).

Also let $Q(1,-2\mu-1,\ \mu)$ be the random point on line (ii).

Then the direction ratios of the line PQ are $\lambda-1,\ -\lambda+2\mu+1,\ \lambda-\mu$.

Since PQ is perpendicular to the line (i), so we have $(\lambda-1).1+(-\lambda+2\mu+1).(-1)+(\lambda-\mu).1=0$

$\Rightarrow 3\lambda-3\mu=2\ \ \ldots$(iii)

Since PQ is perpendicular to the line (ii), so we have $0.(\lambda-1)+(-\lambda+2\mu+1).(-2)+(\lambda-\mu).1=0$

$\Rightarrow 3\lambda-5\mu=2\ \ \ldots$(iv)

Solving (iii) and (iv), we get $\mu=0,\ \lambda=\frac{2}{3}$.

Therefore, the coordinates of point P are $\left(\frac{2}{3},\ -\frac{2}{3},\ \frac{2}{3}\right)$ and that of Q are $(1,-1,\ 0)$.

Hence, the required shortest distance PQ is given by $PQ=\sqrt{\left(1-\frac{2}{3}\right)^2+\left(-1+\frac{2}{3}\right)^2+\left(0-\frac{2}{3}\right)^2}$

$\Rightarrow PQ=\sqrt{\frac{2}{3}}$ units.

SECTION E

36. Let E_1, E_2 and E_3 denote the events that James, Sophia and Oliver processed the form, which are clearly pair wise mutually exclusive and exhaustive set of events.

Then $P(E_1)=\frac{50}{100}=\frac{5}{10}$, $P(E_2)=\frac{20}{100}=\frac{1}{5}$ and $P(E_3)=\frac{30}{100}=\frac{3}{10}$.

Also, let E be the event of committing an error.

We have, $P(E|E_1)=0.06$, $P(E|E_2)=0.04$ and $P(E|E_3)=0.03$.

(i) The probability that Sophia processed the form and committed an error is given by

$$P(E\cap E_2)=P(E_2).P(E|E_2)=\frac{1}{5}\times 0.04$$

$\therefore P(E\cap E_2)=0.008$.

(ii) The total probability of committing an error in processing the form is given by

$$P(E)=P(E_1).P(E|E_1)+P(E_2).P(E|E_2)+P(E_3).P(E|E_3)$$

$$\Rightarrow P(E)=\frac{50}{100}\times 0.06+\frac{20}{100}\times 0.04+\frac{30}{100}\times 0.03$$

$\therefore P(E)=0.047$.

(iii) The probability that the form is processed by James given that form has an error is given by

$$P(E_1|E)=\frac{P(E|E_1)\ P(E_1)}{P(E|E_1)\ P(E_1)+P(E|E_2)\ P(E_2)+P(E|E_3)\ P(E_3)}$$

$$\Rightarrow P(E_1|E)=\frac{0.06\times\frac{50}{100}}{0.06\times\frac{50}{100}+0.04\times\frac{20}{100}+0.03\times\frac{30}{100}}=\frac{30}{47}.$$

Therefore, the required probability that the form is not processed by James given that form has an error $=P(\bar{E}_1|E)=1-P(E_1|E)=1-\frac{30}{47}=\frac{17}{47}$.

OR

(iii) Recall that, the Sum of the posterior probabilities is 1.

So, $\sum_{i=1}^{3}P(E_i|E)=P(E_1|E)+P(E_2|E)+P(E_3|E)=1$

Let's show the proof of above statement.

Consider $P(E_1|E)+P(E_2|E)+P(E_3|E)=\frac{P(E\cap E_1)}{P(E)}+\frac{P(E\cap E_2)}{P(E)}+\frac{P(E\cap E_3)}{P(E)}$

$$\Rightarrow \quad =\frac{P(E\cap E_1)+P(E\cap E_2)+P(E\cap E_3)}{P(E)}$$

$$\Rightarrow \quad =\frac{P\left((E\cap E_1)\cup(E\cap E_2)\cup(E\cap E_3)\right)}{P(E)} \quad \left[\text{as } E_i \text{ and } E_j;\ i\neq j \text{ are mutually exclusive events}\right.$$

$$\Rightarrow \quad =\frac{P\left(E\cap(E_1\cup E_2\cup E_3)\right)}{P(E)}$$

$\Rightarrow \quad = \dfrac{P(E \cap S)}{P(E)} = \dfrac{P(E)}{P(E)} = 1$; where S denote the sample space.

37. We have $\left|\vec{F_1}\right| = \sqrt{6^2 + 0^2} = 6$ kN,

$\left|\vec{F_2}\right| = \sqrt{(-4)^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$ kN,

$\left|\vec{F_3}\right| = \sqrt{(-3)^2 + (-3)^2} = \sqrt{18} = 3\sqrt{2}$ kN.

(i) Magnitude of force of Team A = 6 kN.

(ii) Since, 6 kN is largest so, team A will win the game.

(iii) As $\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} = 6\hat{i} + 0\hat{j} - 4\hat{i} + 4\hat{j} - 3\hat{i} - 3\hat{j} = -\hat{i} + \hat{j}$

$\therefore \left|\vec{F}\right| = \sqrt{(-1)^2 + (1)^2} = \sqrt{2}$ kN.

OR

(iii) As $\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} = -\hat{i} + \hat{j}$

To find the direction in which the ring is getting pulled, we shall find the angle of resultant force $\vec{F}$ with the x-axis.

Note that the direction ratios of x-axis are 1, 0, 0.

Also for $\vec{F}$, the direction ratios are $-1, 1, 0$.

$\therefore \cos\theta = \dfrac{1(-1) + 0(1) + 0(0)}{\sqrt{1^2 + 0^2 + 0^2}\sqrt{(-1)^2 + 1^2 + 0^2}} = -\dfrac{1}{\sqrt{2}} \qquad \Rightarrow \theta = \cos^{-1}\left(-\dfrac{1}{\sqrt{2}}\right)$

$\therefore \theta = \pi - \cos^{-1}\left(\dfrac{1}{\sqrt{2}}\right) = \pi - \dfrac{\pi}{4} = \dfrac{3\pi}{4}$ [where θ is the angle made by the resultant force with the positive direction of the x-axis]

38. Given that $y = 4x - \dfrac{1}{2}x^2$

(i) Rate of growth of the plant with respect to the number of days exposed to sunlight is given by $\dfrac{dy}{dx} = 4 - x$.

(ii) Let rate of growth be represented by the function $g(x) = \dfrac{dy}{dx}$.

Now, $g'(x) = \dfrac{d}{dx}\left(\dfrac{dy}{dx}\right) \qquad \Rightarrow g'(x) = \dfrac{d}{dx}(4 - x) = -1$

$\therefore g'(x) = -1 < 0$

$\Rightarrow g(x)$ decreases.

So the rate of growth of the plant decreases for the first three days.

Height of the plant after 2 days is given by $y = 4 \times 2 - \dfrac{1}{2}(2)^2 = 6$ cm.